Chapter 3 Part 1 Notes
3.1 – Factors and Multiples

Goal: to determine prime factors, greatest common factors, and least common multiples of whole numbers

Toolkit:
- Division
- Multiplication
- Writing repeated multiplication using powers,
  e.g. \(2 \times 2 \times 2 \times 2 \times 2 = 2^6\)

Main Ideas:

Definitions

Factor – a term which divides evenly into another term

Prime number – when a number has only 2 distinct factors (1 and itself). Examples: 2, 3, 5, 7, 11, 13, 17, 19, 23

Composite number – when a number has more than 2 factors. Examples: 4, 6, 8, 9 = 3 \times 3, 14 = 7 \times 2

Prime factorization – a term written as a product of prime factors

*every composite number can be expressed as a product of prime factors*

Greatest common factor (GCF) = largest term which will divide evenly into a series of separate terms

Least (or Lowest) common multiple (LCM) – the smallest multiple which is common to series of separate terms

Prime Factorization

Ex1) Write the prime factorization for each of the composite numbers:

\(9, 12, 45, 3300\)

\(9 = 3 \times 3\)

\(12 = 2 \times 2 \times 3\)

\(45 = 3 \times 3 \times 5\)

\(3300 = 2 \times 2 \times 3 \times 5 \times 5 \times 11\)

Finding the GCF

by listing all the factors of each number (the rainbow method)

Ex2) Determine the greatest common factor of 126 and 144

Method 1 – list all the factors and find the largest one in common

\(126 = 2 \times 3 \times 3 \times 7\)

\(144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3\)

Greatest common factor is \(18\)

Method 2

1) write the prime factorization for each number
2) highlight the factors that they have in common
3) multiply all the common factors together go get the GCF

\(126 = 2 \times 3 \times 3 \times 7\)

\(144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3\)

Greatest common factor is \(18\)
Ex3) Find the least common multiple of 28, 42, and 63

**Method 1** – list the first few multiples of each number until you find the first, lowest one in common

- Multiples of 28: 28, 56, 84, 112, 140, 168, 196, 224, 252
- Multiples of 42: 42, 84, 126, 168, 210, 252
- Multiples of 63: 63, 126, 189, 252

Lowest common multiple doesn’t show up until 252!

**Method 2**
1) write the prime factors of each number
2) highlight the greatest power of each prime in ANY of the lists
3) multiply the greatest powers of each prime together to get the LCM

\[
\begin{align*}
28 &= 2^2, 7 \\
42 &= 2^1, 3, 7 \\
63 &= 3^2, 7 \\
\text{LCM} &= 2^2, 3^2, 7 \\
&= 252
\end{align*}
\]

check 1: \(2^2\) is highest power of 2. check 2: \(3^2\) is highest power of 3. check 7: \(7\) is highest power of 7. no other prime factors

Ex4) Beside each problem, write whether you would need the GCF or the LCM, then answer the question!

a) A bathroom wall (the part above the bathtub) is a rectangle that measures 78" by 60". If you wanted to cover it exactly with square tiles, what is the largest possible square tile you could use?

\[
\begin{align*}
&\text{60 = } 2^2 \cdot 3 \cdot 5 \\
&\text{78 = } 2 \cdot 3^2 \cdot 13 \\
\text{GCF = } 2 \cdot 3 \\
\end{align*}
\]

The largest square tile you could use are 6" by 6".

b) You have red bungee cords that are 18cm long and green bungee cords that are 14cm long. What is the shortest length of connected bungees you can make with each colour so that they make the same length?

\[
\begin{align*}
\text{LCM} &= 2^1 \cdot 3^2 \cdot 7 \\
&= 252
\end{align*}
\]

The shortest length you could make would be 126cm.

**Reflection:** How can you remember the difference between a factor and a multiple? Write (or make) a memory trick to help you.

"Factors" are pieces

"Multiples" is multiply (gets bigger)
3.2 – Perfect Squares, Perfect Cubes, and their Roots

Goal: to identify perfect squares and perfect cubes, and to find square roots and cube roots

Toolkit:
- Prime factorization – no calculator!
- The opposite operation of squaring is the square root:
  \[ 5^2 = 25 \text{ and } \sqrt{25} = 5 \]
- The opposite operation of cubing is the cube root:
  \[ 2^3 = 2 \times 2 \times 2 = 8 \text{ and } 3\sqrt{8} = 2 \]

Main Ideas:

What is a Perfect Square?

A perfect square is a number that can be written as the product of two equal factors.

This means you can represent it as the AREA OF A SQUARE! \( A = b \times b = b^2 \)

Picture an actual square! The square root is the side length of the square

Ex1) Determine the square root of 1296.
Step 1: Write 1296 as a product of its prime factors

\[ 1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \]

Step 2: Re-order the prime factors into TWO identical groups. (If you can’t, your number is NOT a perfect square).

\[ (2 \times 2 \times 3 \times 3) (2 \times 2 \times 3 \times 3) \]

Step 3: Multiply out each "group" again to see what number it represents

\[ 36 \times 36 \]

Since 1296 can be written as the product of TWO equal factors: \( 36 \times 36 \), it can be represented as the area of a square.

The square root of 1296 is \( 36 \).

We write \( \sqrt{1296} = 36 \)

Terminology: radical, radicand, index:
**What is a Perfect Cube?**

A perfect cube is a number that can be written as the product of 3 equal factors.

This means you can represent it as the VOLUME OF A CUBE: \( V = e \times e \times e = e^3 \)

**Picture an actual cube!**

\[ \sqrt[3]{2 \times 2 \times 2} = 2 \]

**Cube root of 8 is 2.**

**Determining a Cube Root**

Ex2) Determine the cube root of 1728.

Step 1: Write 1728 as a product of its prime factors.

\[ 1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \]

\[ 2 \times 2 \times 2 \times (2 \times 2) \times (2 \times 3) \]

Step 2: Re-order the prime factors into THREE identical GROUPS. (If you can't, your number is NOT a perfect cube).

\[ (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 3 \times 3) \]

Step 3: Multiply out each "group" again to see what number it represents.

\[ (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 3 \times 3) = 12 \times 12 \times 12 \]

Since 1728 can be written as the product of THREE equal factors: \( 12 \times 12 \times 12 \), it can be represented as the volume of a cube.

The cube root of 1728 is \( \sqrt[3]{1728} = 12 \).

**We write**

\[ \sqrt[3]{1728} = 12 \]

**Extend your thinking:**

Ex3) Determine the edge length of a cube with volume 64\(x^6\).

\[ V = (4x^2)^3 \]

\[ \text{edge length} = 4x^2 \]

**Reflection:** How could you ESTIMATE the square root or cube root of a number? (Think back to math 9?)

Use square roots of perfect squares.

ex: \( \sqrt{5} \) is between \( \sqrt{4} \) and \( \sqrt{9} \).
Goal: to expand monomial and binomial products (multiply out!)

Toolkit:
- Adding, subtracting, multiplying polynomials
- Multiplying powers with the same base: add the exponents
  \[ (x^a x^b) = x^{a+b} \]
- Collecting like terms: same variable(s) with same exponents

\[ (2x^2 + 3x - x^2 + 2x + 1) = 1x^2 + 5x + 1 \]

Main Ideas:

Definitions

**Polynomial** - poly = many terms (terms separated by add, subtract)

**Monomial** - one term

**Binomial** - 2 terms

**Trinomial** - 3 terms

\[ e.g. \quad 3x^2y^3 \quad (2x^2y + 3xy) \quad (x^2 + 3x + 2) \]

Ex1) Expand and simplify \( \rightarrow \) translates to: multiply out and collect like terms

a) \[ 3x^2(x + 3) \]

\[ 3x^3 + 9x^2 \]

Think: \( (3x^2)(x) \) and \( (3x^2)(3) \)

\[ 3x^3 \quad \text{and} \quad 9x^2 \]

b) \[ (x + 2)(x + 3) \]

\[ x^2 + 3x + 2x + 6 \]

\[ x^2 + 5x + 6 \]

Stay organized!
- First in each bracket: \( x \cdot x = x^2 \)
- Outside: \( x \cdot 3 = 3x \)
- Inside: \( 2 \cdot x = 2x \)
- Last in each bracket: \( 2 \cdot 3 = 6 \)

**FOIL**

Ist, Outside, Inside, Last
...Expand and simplify

c) \((2y + z)(3y - 2z)\)

\((2y)(3y) + (2y)(-2z) + (z)(3y) + (z)(-2z)\)

\(6y^2 - 4yz + 3yz - 2z^2\)

\(6y^2 - yz - 2z^2\)

d) \((2a - 1)(2a + 3) + (a - 1)(a - 2)\)

\((4a^2 + 6a - 2a - 3) + (a^2 - 2a - 1a + 2)\)

\(4a^2 + 4a - 3 + a^2 - 3a + 2\)

\(5a^2 + a - 1\)

Ex2) Find the area of the shaded region (simplified!): \[ A_{\text{shaded}} = A_{\text{big rect}} - A_{\text{little rect}} \]

\[
\begin{array}{c}
6x + 1 \\
2x + 4
\end{array}
\]

\[
\begin{array}{c}
3x \\
x + 2
\end{array}
\]

A_{\text{big rect.}} = l \times w

\((2x+4)(6x+1)\)

\(12x^2 + 2x + 24x + 4\)

\(12x^2 + 26x + 4\)

A_{\text{little rect.}} = l \times w

\(3x(x+2)\)

\(3x^2 + 6x\)

\[ A_{\text{shaded}} = (12x^2 + 26x + 4) - (3x^2 + 6x) \]

Shaded area: \(9x^2 + 20x + 4\)

Reflection: Why can you only collect LIKE TERMS? You may wish to use an example to help you explain.

You can't add apples and oranges! \(2x + 3x = 5x\)

\(28 + 50 = 28 + 50!\)

\(28 + 38 = 58\)
3.7B – Multiplying Polynomials

Goal: to expand and simplify polynomials with more than 2 terms

Toolkit:
- FOIL
- Adding, subtracting, multiplying polynomials
- Multiplying powers with the same base: add the exponents
- Collecting like terms: same variable(s) with same exponents

Main Ideas:

Ex1) Expand and simplify:

a) 
\((x + 3y)(x + y - 3)\)

\[x^2 + xy - 3x + 3xy + 3y^2 - 9y\]
\[x^2 - 3x + 4xy + 3y^2 - 9y\]

b) 
\((x + 2)^3\)

- Multiply base by itself 3 times!
- For one pair
  \((x + 2)(x + 2)(x + 2)\)
- Simplify

\((x + 2)(x^2 + 2x + 2x + 4)\)

Line it up cleverly:
\[x^3 + 4x^2 + 4x + 2x^2 + 8x + 8\]
\[x^3 + 6x^2 + 12x + 8\]

Note: for questions like \(3x^2(x+1)(x+2)\), FOIL first!

3.7B – Multiplying Polynomials

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- Collecting like terms: same variable(s) with same exponents

Main Ideas:

Ex1) Expand and simplify:

a) 
\((x + 3y)(x + y - 3)\)

\[x^2 + xy - 3x + 3xy + 3y^2 - 9y\]
\[x^2 - 3x + 4xy + 3y^2 - 9y\]

b) 
\((x + 2)^3\)

- Multiply base by itself 3 times!
- For one pair
  \((x + 2)(x + 2)(x + 2)\)
- Simplify

\((x + 2)(x^2 + 2x + 2x + 4)\)

Line it up cleverly:
\[x^3 + 4x^2 + 4x + 2x^2 + 8x + 8\]
\[x^3 + 6x^2 + 12x + 8\]

Note: for questions like \(3x^2(x+1)(x+2)\), FOIL first!