Chapter 3 Part 2 Notes

*STUDENT COPY*

<table>
<thead>
<tr>
<th>Marks \ Requirement ↓</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Notes Present</td>
<td>All notes present</td>
<td>Most notes present</td>
<td>Less than half of notes present</td>
</tr>
<tr>
<td>Organization / Neatness</td>
<td>Notes in chronological order, name and date on everything</td>
<td>Almost all notes in chronological order, name and date on most pages</td>
<td>Mostly out of order, name and date often missing</td>
</tr>
<tr>
<td>Questions</td>
<td>Question column completed on all notes, higher level questions attempted</td>
<td>Most question columns complete, some higher level questions</td>
<td>Less than half of the question columns complete</td>
</tr>
<tr>
<td>Main Ideas and Reflections</td>
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<td>Less than half of the ‘main ideas’ and ‘reflections’ complete</td>
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*If your mark does not total up to at least 4 out of 8, your notes are INCOMPLETE and must be fixed up as soon as possible and re-evaluated.

*TEACHER COPY*

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**3.7 – ALGEBRA TILES**

**Goal:** to model polynomial products using algebra tiles

How do we use algebra tiles?

<table>
<thead>
<tr>
<th>Positive (yellow)</th>
<th>Negative (red)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x²</td>
<td>x</td>
</tr>
</tbody>
</table>

Model: 

\((x + 1)(x + 2)\)

Make the LENGTH of the rectangle \(x + 2\) and the WIDTH of the rectangle \(x + 1\). Fill in the rectangle, and see how many pieces (and what kinds) you need to do so.

**Total Area is** \(x² + 3x + 2\)

Ex3) Model the following products from using algebra tiles: *(Sketch!)*

a) \((x + 4)(x + 1)\)

\[x² + 5x + 4\]

b) \((x + 3)(x - 2)\)

\[x² + 3x - 2x - 6\]

\[x² + x - 6\]

**Reflection:** How (if at all) do the algebra tiles help you picture the multiplication?
# 3.3 – Common Factors of a Polynomial

**Goal:** to determine the factors of a polynomial by identifying the GCF

### Toolkit:
- Finding the GCF
- Distributive Property

### Main Ideas:

| Factor a binomial using the GCF | Ex 1) Factor the binomial $3g + 6$
<table>
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<tbody>
<tr>
<td>[3(g + 2)]</td>
<td>What is the GCF of $3g$ and $6$? GCF = 3</td>
</tr>
<tr>
<td></td>
<td>(so factor 3 out of each term)</td>
</tr>
<tr>
<td>Check using distributive : $3(g+2)$ = $3g + 6$ ✓</td>
<td></td>
</tr>
</tbody>
</table>
| Ex 2) Factor the binomial $-8y + 16y^2$
| GCF of $-8y$ and $16y^2$? GCF = $8y$ |
| $8y(-1+2y)$                      | Check: $8y(-1+2y) = -8y + 16y^2$ ✓ |

<table>
<thead>
<tr>
<th>Factor a trinomial using the GCF</th>
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</table>
| Ex 3) Factor the trinomial $3x^2 + 12x - 6$
| GCF of $3x^2$, $12x$ and $-6$? GCF = 3 |
| $3(x^2 + 4x - 2)$               |
| Check: $3(x^2 + 4x - 2)$ = $3x^2 + 12x - 6$ ✓ |

<table>
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<tr>
<th>Factor polynomials in more than one variable</th>
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</table>
| Ex 4) Factor the trinomial $6 - 12z + 18z^2$
| GCF = 6                                       |
| $6(1 - 2z + 3z^2)$                          |
| Check: $6 - 12z + 18z^2$ ✓                   |

| Ex 5) Factor the trinomial $-20c^4d - 30c^3d^2 - 25cd$
| GCF of $-20c^4d$, $-30c^3d^2$, $-25cd$? GCF = $5cd$ |
| $5cd(-4c^3d - 6c^2d - 5)$                   |
| Check: $-20c^4d - 30c^3d^2 - 25cd$ ✓        |

Could we have factored out a "-5cd"? We usually want the first term to be positive.

### Reflection:
How are the processes of factoring and expanding related?

They are inverses of each other.
3.5 – Factoring Trinomials of the form $x^2 + bx + c$, where $a=1$

**Goal:** to use models and algebraic strategies to multiply binomials and to factor trinomials.

**Toolkit:**
- Factoring $ax^2 + bx + c$
- $a$ with $x$
- $x^2$

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**Definitions:**
- **Descending order:** the terms are written in order from the term with the greatest exponent to the term with the least exponent
- **Ascending order:** the terms are written in order from the term with the least exponent to the term with the greatest exponent

**Steps for Factoring a Trinomial in the form $x^2 + bx + c$, where $a=1$**
- **Step 1:** If needed, re-order the terms in descending powers of the variable (biggest to smallest)
- **Step 2:** Find two numbers that multiply to equal the $c$ term and add to equal the $b$ term (add to the middle, multiply to the end)
- **Step 3:** Factor into two binomials using the numbers from step 2, with the variable from the question placed first in each bracket

**Example 1:** Expand and Simplify: $(x - 1)(x - 7)$$\Rightarrow$ use FOIL

- $x^2 - 7x - x + 7 = x^2 - 8x + 7$
- $b = -8$ came from adding
- $-7x$ and $-1x$
- $c = 7$ came from multiplying $(-1)(-7)$

Remember: expanding and factoring are opposite operations...they UNDO each other!

**Example 2:** Factor the trinomial: $x^2 - 8x + 7$

- $\text{Re-order } \checkmark$
- $\text{Need } s \text{ that multiply to } c = 7$ and add to $b = -8$
- $1 \times 7 \Rightarrow 1 + 7 = 8$
- $-1 \times -7 \Rightarrow -1 + (-7) = -8$ $\checkmark$

**Example 3:** Factor: $a^2 - 2a - 8$
- $\text{GCF? } \checkmark$
- $\text{Re-order } \checkmark$
- $\text{Need } s \text{ that multiply to } c = -8$ and add to $b = -2$
- $-1 \times 8 \Rightarrow -1 + 8 = 7$
- $-2 \times 4 \Rightarrow -2 + 4 = 2$
- $2 \times -4 \Rightarrow 2 - 4 = -2$ $\checkmark$

**Example 4:** Factor: $-30 + 7m + m^2$
- $\text{GCF? } \checkmark$
- $\text{Re-order } \text{ } a^2 \ 7 \ 30$
- $\text{To } -30$ $\text{To } 7$
- $+3 \times -10 \Rightarrow -3 \times 10 \Rightarrow +7$ $\checkmark$
- $\text{Check: } m^2 + 10m - 3m - 30$
- $m^2 + 7m - 30$ $\checkmark$
Ex 5) Factor: \(-5h^2 - 20h + 60\)

First, factor out the GCF if there is one. If there is a negative number in front of the \(x\), factor out the negative as well.

\[ -5 \left( h^2 + 4h - 12 \right) \]

\[ -5 (h - 2)(h + 6) \]

Check: For first,
\[ -5 \left( h^2 + 6h - 2h - 12 \right) \]
\[ -5 \left( h^2 + 4h - 12 \right) \]
\[ -5h^2 - 20h + 60 \]

Circle answer!

Ex 6) Factor: \(-12 - 9g + 3g^2\)

Recall the GCF:
\[ 3(g^2 - 3g + 4) \]

Check: For first,
\[ 3(g^2 - 4g + 1g - 4) \]
\[ 3(g^2 - 3g - 1) \]
\[ 3g^2 - 9g - 12 \]

Circle answer!

Ex 7) Factor: \(2x^2 - 6x - 80\)

Recall? GCF?

\[ 2(x^2 - 3x - 40) \]

Check: For first,
\[ 2(x^2 + 5x - 8x - 40) \]
\[ 2(x^2 - 3x - 40) \]
\[ 2x^2 - 6x - 80 \]

Circle answer!

Ex 8) Factor: \(x^2 + 4x - 2\)

\[ (x+1)(x+2) \]

\[ = x^2 + 2x + x - 2 \]
\[ = x^2 + x - 2 \]

\[ (x+2)(x-1) \]

\[ = x^2 + 2x + 2x - 2 \]
\[ = x^2 + x - 2 \]

Reflection: Does the order in which the binomial factors are written affect the solution? Explain.

No! You get the same product regardless of the order you multiply in.
3.6 – Polynomials of the Form \( ax^2 + bx + c, \ a \neq 1 \)

**Goal:** to extend the strategies for multiplying binomials and factoring trinomials

<table>
<thead>
<tr>
<th>Toolkit:</th>
<th>Main Ideas:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Multiplying binomials</td>
<td></td>
</tr>
<tr>
<td>• Factoring</td>
<td></td>
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</table>

**Factoring by Decomposition:** (needed when the \( a \neq 1 \) in \( ax^2 + bx + c \))

With any factoring question, first check to see if you can factor out a GCF from ALL terms!

Step 1: If needed, re-order the terms in descending powers of the variable (biggest to smallest)
Step 2: Find two numbers that multiply to equal \( ac \) and add to equal \( b \) (add to the middle, multiply to product of first and last)
Step 3: Re-write the expression but split or decompose the \( b \) term using the two numbers from step 2.
Step 4: Now the expression has FOUR terms, so we can factor by grouping the first two terms and the last two terms.
Step 5: When fully factored, the remaining two brackets need to be identical! These are now a common factor, and can be factored out, and what is left becomes the components of the second bracket.

### Factor by Grouping

| Ex. 1) Factor the following by grouping: |
|---|---|
| a) \( 3x^2 - 3x - 2x + 2 \) |
| gcf: | gcf: 2 |
| \( 3x \) | \( 2 \) |
| \( 3x(x-1) - 2(x-1) \) | \( 2 \) |
| \( (x-1) (3x-2) \) | \( (x-1) (2x+1) \) |
| Can check! | |

### Factoring a trinomial of the form \( ax^2 + bx + c \)

Notice that \( a \) (the number in front of \( x^2 \)) is not \( 1 \) in any of these questions!

### Ex 2) Factor the trinomial: \( 4x^2 + 11x + 6 \) by decomposition

\( a: 4, \ b: 11, \ c: 6 \)

1. Take out common "bracket" 
2. Factor the trinomial: 
3. 4g2 + 11g + 6 
4. Need to  
5. 6 to 24  
6. 4 + 11 
7. 1 x 24  
8. 1 x 24  
9. 2 x 12  
10. 3 x 8  
11. 3 x 11  
12. (x-1) (3x-2)  
13. (x-2) (2x+1)  
14. Check:

### Ex 3) Factor the trinomial: \( -7m - 10 + 6m^2 \)

1. Factor the trinomial: 
2. \( 6m^2 - 7m - 10 \) 
3. \( 6m^2 + 5m - 12m - 10 \) 
4. \( (m+2)(3m-5) \) 
5. \( (m+5)(m-2) \)
Ex 4) Factor: \(8p^2 - 18p - 5\)
\[\begin{align*}
\text{GCF: 2} & \quad 40 + -18 \\
\text{Factors: 2 and 20} & \quad -18 \\
\text{Check:} & \quad -8p^2 - 20p + 25 \checkmark
\end{align*}\]

Ex 5) Factor: \(6x^2 + 14x - 12\)
\[\begin{align*}
\text{GCF: 2} & \quad 12 + -18 \\
\text{Factors: 2 and 6} & \quad 7 \\
\text{Check: FOIL first} & \quad 2(3x^2 - 2x + 9x - 6) \\
& \quad 2(3x^2 + 7x - 6) \\
& \quad 6x^2 + 14x - 12 \checkmark
\end{align*}\]

Ex 6) Factor: \(3x^2 + 6x - 9\)
\[\begin{align*}
\text{Factors: 3 and 3} & \quad 2 \\
\text{Check:} & \quad 3(x^2 + 2x - 3) \\
& \quad 9x^2 - 1x + 3 \not\checkmark
\end{align*}\]

Ex 7) Find an integer to replace \(\Box\) so that the trinomial can be factored. How many integers can you find?
\[\begin{align*}
4x^2 + \Box x + 9 & \quad \rightarrow 20 \\
\text{must \ GCF \ 36!} & \quad \rightarrow -20 \\
3x^2 + \Box x + 6 & \quad \rightarrow 15 \\
\text{GCF: 3} & \quad \rightarrow -15
\end{align*}\]
10 different answers!

Reflection: Will decomposition work if the \(a\) value of a trinomial is 1? Do an example to prove this.

Yes!
\[\begin{align*}
\frac{1}{3}x^2 + \frac{5}{3}x + 6 \quad \rightarrow 6x1 = 6 \quad \rightarrow 5 \\
2x^2 + 6x + 6 \quad \rightarrow 2x3 = 3 + 3 \\
(x+2) + 3(x+2) \\
(x+2)(x+3)
\end{align*}\]
3.8 – Factoring Special Polynomials

Goal: to investigate some special factoring patterns

Toolkit:
- Finding a square root
- Finding GCF
- Multiplying Polynomials

Main Ideas:

Definitions:

Perfect Square Trinomial: a trinomial of the form \( a^2 + 2ab + b^2 \); it can be factored as \((a + b)^2\)

or of the form \( a^2 - 2ab + b^2 \); it can be factored as \((a - b)^2\)

Difference of Squares: a binomial of the form \( a^2 - b^2 \); it can factored as \((a - b)(a + b)\)

Draw an area model of a square with side length \( a + b \) to represent a perfect square trinomial.

\[
\begin{array}{|c|c|}
\hline
a & b \\
\hline
a & a^2 & ab \\
\hline
b & ab & b^2 \\
\hline
\end{array}
\]

Area = \( a^2 + ab + ab + b^2 = a^2 + 2ab + b^2 \)

This is a perfect square trinomial

\[
a^2 + 2ab + b^2 \quad \text{factors to} \quad (a+b)^2
\]

\[
a^2 - 2ab + b^2 \quad \rightarrow \quad (a-b)^2
\]

Ex 1) Factor the trinomial: \( 36x^2 + 12x + 1 \) Any common factors?

No

Ex 2) Factor the trinomial: \( 5c^2 - 13cd + 6d^2 \) Any common factors?

No
Ex 3) Factor the binomial: $81m^2 - 49$

- perfect square:
  - $\sqrt{81} = 9m$
  - $\sqrt{49} = 7$

$$= (9m - 7)(9m + 7)$$

- $(a-b)(a+b)$

Ex 4) Factor the binomial: $162v^2 - 2w^4$

- gcf: $2^1$

$$= 2(81v^2 - w^4)$$

$$= 2(9v - w^2)(9v + w^2)$$

- Can you factor further? No.

Ex 5) Factor the binomial: $\frac{x^2}{25} - \frac{y^2}{4}$

$$= \left(\frac{x}{5}\right)^2 - \left(\frac{y}{2}\right)^2$$

$$= \left(\frac{x}{5} - \frac{y}{2}\right)\left(\frac{x}{5} + \frac{y}{2}\right)$$


Try $(6n+2)(6n+2) = 36n^2 + 12n + 12n + 4$

$= 36n^2 + 24n + 4$
3.9 – Factoring Synthesis

FACTORING FLOW CHART

STEP 1  Take out COMMON FACTORS (GCF)

STEP 2  Ask: How many terms are there?

TWO

Probably a difference of squares:
*You need subtraction ("difference") and squares

\[ a^2 - b^2 = (a + b)(a - b) \]

**Diff of Sqs = Conjugates**

Example:

\[ 4x^2 - 9 = \]

\[ (2x)^2 - (3)^2 = \]

\[ (2x + 3)(2x - 3) \]

THREE

Factoring trinomials:
\[ ax^2 + bx + c \]

Type 1: \( a = 1 \)

Example:

\[ x^2 - 3x + 2 \]

Ask: what ADDS to "b" (here -3)

& MULTIPLIES to "c" (here +2)

Answer: \(-1, -2\)

Write factors

\[ (x - 1)(x - 2) \]

Type 2: \( a \neq 1 \)

Example:

\[ 2x^2 - x - 1 \]

Ask: what ADDS to "b" (here -1)

& MULTIPLIES to "ac" (here \(2(-1) = -2\))

Answer: \(-2, 1\)

Use these to split the middle term into two separate terms:

\[ 2x^2 - x - 1 \]

\[ 2x^2 - 2x + 1x - 1 \]

Factor using grouping:

See next column ☞

FOUR

Probably grouping:

Example:

\[ 2x^2 - 2x + 1x - 1 \]

Group the first two terms together, and the last two terms together:

\[ [2x^2 - 2x] + [1x - 1] \]

Factor common factors out of each group:

\[ 2x(x - 1) + 1(x - 1) \]

You should have two matching brackets.

Factor them out:

\[ (x - 1)(2x + 1) \]

STEP 3  Ask: FF? Look inside each factor (bracket) and see if you can FACTOR FURTHER.