# Chapter 5 Notes

**STUDENT COPY**

<table>
<thead>
<tr>
<th>Marks Requirement</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Notes Present</td>
<td>All notes present</td>
<td>Most notes present</td>
<td>Less than half of notes present</td>
</tr>
<tr>
<td>Organization / Neatness</td>
<td>Notes in chronological order, name and date on everything</td>
<td>Almost all notes in chronological order, name and date on most pages</td>
<td>Mostly out of order, name and date often missing</td>
</tr>
<tr>
<td>Questions</td>
<td>Question column completed on all notes, higher level questions attempted</td>
<td>Most question columns complete, some higher level questions</td>
<td>Less than half of the question columns complete</td>
</tr>
<tr>
<td>Main Ideas and Reflections</td>
<td>All ‘main ideas’ and ‘reflections’ complete with care in notes</td>
<td>Most ‘main ideas’ and ‘reflections’ complete in notes</td>
<td>Less than half of the ‘main ideas’ and ‘reflections’ complete</td>
</tr>
</tbody>
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*If your mark does not total up to at least 4 out of 8, your notes are INCOMPLETE and must be fixed up as soon as possible and re-evaluated.

**TEACHER COPY**

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5.1 – Representing Relations

Goal: to discuss the concept of a relation and to represent relations in different ways

Toolkit:
- anything you know about “relations”
- “” “” “” graphs
- word math

Main Ideas:

Definitions:

Set – a set is a collection of distinct objects

Element – an element of a set is one object in the set

Relation – a relation associates the elements of one set with the elements of another set

There are many ways to represent a relationship between two sets. Be prepared to recognize these terms and match them to the different representations:

Words, Table, Diagram, Arrow Diagram, Bar Graph, Ordered Pairs, (Line Graph) Later

Ex1) When we talk about a Gulf Islands community, we may want to know on which island it is located.

<table>
<thead>
<tr>
<th>Community</th>
<th>Gulf Island</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fulford Harbour</td>
<td>Salt Spring Island</td>
</tr>
<tr>
<td>Gillies Bay</td>
<td>Texada Island</td>
</tr>
<tr>
<td>Sturdies Bay</td>
<td>Galiano Island</td>
</tr>
<tr>
<td>Long Harbour</td>
<td>Salt Spring Island</td>
</tr>
<tr>
<td>Blubber Bay</td>
<td>Texada Island</td>
</tr>
<tr>
<td>Vesuvius</td>
<td>Salt Spring Island</td>
</tr>
</tbody>
</table>

a) What type of relation is presented?
   a table

b) Describe the relation in words
   community "is located on" island

c) Represent the relation as an arrow diagram

d) Represent the relation as a set of ordered pairs
   (Fulford H, SSpring), (Gillies B, Tex.), (Sturdies B, Gal.), (Long H, SSpring), (Blubber B, Tex.), (Vesuvius, SSpring).

Note: could this be made into a bar graph? No - need numbers somewhere.
Ex2) This **bar graph** shows the relationship between different breeds and their mean (average) heights.

Represent this relation
a) In words
b) As a table
c) As an arrow diagram

\[ \text{has a mean height of} \]

\[
\begin{array}{|l|c|}
\hline
\text{breed} & \text{mean height} \\
\hline
\text{Afghan} & 75 \\
\text{Chihuahua} & 20 \\
\text{Corgi} & 30 \\
\text{Golden Ret.} & 60 \\
\text{German Sh.} & 60 \\
\text{Malamute} & 65 \\
\hline
\end{array}
\]

*Note: Arrow diagram - as are smallest to largest; no repeats*

Ex3) In the **diagram**,
a) Describe the relation in words
b) List 2 ordered pairs that belong to the relation

\[ \text{this word} \]

\[ \text{has this number of letters in it} \]

\( (\text{denominator}, 11), (\text{polynomial}, 10) \)

*etc.*

---

**Reflection:** Which method of representing a relation makes the most sense to you? Why? List its advantages and disadvantages. 
5.2A – Properties of Functions

Goal: to develop the concept of a function and to be able to recognize functions

Toolkit:
- relations
- arrow diagrams, etc. (5.1)
- variables

Main Ideas:

Definitions
Domain – The set of first elements of a relation is called the domain

Range – The set of second elements of a relation is called the range

Function – A function is a special type of relation where each element in the domain is associated with exactly one element in the range (OR a set of ordered pairs in which no two ordered pairs have the same first co-ordinate)

Independent Variable – An independent variable is a variable whose value is not determined by the value of another variable

Dependent Variable – A dependent variable is a variable whose value is determined by the value of another (the independent) variable

Ex1) State the domain and range for each relation:

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Animal</td>
<td># of legs</td>
</tr>
<tr>
<td>Chicken</td>
<td>2</td>
</tr>
<tr>
<td>Dog</td>
<td>4</td>
</tr>
<tr>
<td>Cat</td>
<td>4</td>
</tr>
<tr>
<td>Spider</td>
<td>8</td>
</tr>
<tr>
<td>Ladybug</td>
<td>6</td>
</tr>
<tr>
<td>Eagle</td>
<td>2</td>
</tr>
</tbody>
</table>

Domain: \{chicken, dog, cat, spider, ladybug, eagle\}
Range: \{2, 4, 6, 8\}

Note: don’t need to repeat!

Ex1) (a) Domain: \{1, 4, 9\}
Range: \{3, -3, -2, -1, 1, 2, 3\}

(b) Domain: \{(x, y)\}
Range: \{(1, 3), (4, -2), (9, 2)\}

(c) \{(-2, 4), (-1, 1), (1, 1), (2, 4), (3, 9)\}
Domain: \{-2, -1, 1, 2, 3\}
Range: \{1, 4, 9\}
How do we determine whether a relation is also a function?

For a table of values or ordered pairs: NO REPEATS of domain (1st) elements allowed. 2nd elements can repeat. Each domain must match up with exactly one range element.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Animal</td>
<td># of legs</td>
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<tr>
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</tr>
<tr>
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<td>4</td>
</tr>
<tr>
<td>Cat</td>
<td>2</td>
</tr>
<tr>
<td>Spider</td>
<td>8</td>
</tr>
<tr>
<td>Ladybug</td>
<td>6</td>
</tr>
<tr>
<td>Eagle</td>
<td>2</td>
</tr>
</tbody>
</table>

\( \{ (2,4), (-1,1), (1,1), (2,4), (3,9) \} \)

\( \boxed{\text{No repeats of 1st element: is a function}} \)

\( \{ (1,1), (1,2), (3,3), (3,4) \} \)

\( \boxed{\text{Not a function}} \)

For an arrow diagram:

For an arrow diagram:

Only ONE arrow can leave each entry in the left (domain) bubble. Many arrows may arrive at an entry on the right.

\( \boxed{\text{Not a function}} \)

Ex2) Students are doing a “nickel drive” fund raiser. The amount of money they raise will depend on the number of nickels turned in.

a) label the domain/range, independent/dependent variables

b) is this relation a function, or not a function?

\[
\begin{array}{|c|c|}
\hline
\text{Number of nickels, } n & \text{Amount raised, } A (\$) \\
\hline
0 & 0 \\
50 & 2.50 \\
100 & 5.00 \\
150 & 7.50 \\
200 & 10.00 \\
\hline
\end{array}
\]

A depends on \( n \)

Some range values (there are more!)

Would this pattern continue? Yes!

This is a function. If you tell me the # nickels raised, I can tell you exactly the amount raised.

1 nickel raised is only ever worth $0.05!

Reflection: Complete the Frayer model on the next page.
Goal: to represent information about function in a Frayer model (refer to page 274 for ideas)

Reflection: How does this model help you? What works for you? What doesn't?
### 5.2B – Function Notation

**Goal:** to define and work with function notation

#### Toolkit:
- functions
- substitution
- solving for a variable (rearranging)

#### Main Ideas:

<table>
<thead>
<tr>
<th>Toolkit:</th>
<th>Main Ideas:</th>
</tr>
</thead>
<tbody>
<tr>
<td>- functions</td>
<td>Ways to think about functions:</td>
</tr>
<tr>
<td>- substitution</td>
<td>- rules</td>
</tr>
<tr>
<td>- solving for a variable (rearranging)</td>
<td>- formulas</td>
</tr>
<tr>
<td></td>
<td>- input/output machines</td>
</tr>
</tbody>
</table>

A domain value goes IN, then the function machine changes it, and the (one and only) matching range value comes OUT.

Recall the "nickel drive" fund raiser. What does the machine do? \( x \times 0.05 \)

**Account for:** independent/dependent, domain/range, input/output, the variables

![Diagram](image)

Function notation shows us mathematically that the Amount of money raised (A) depends on (is a function of) the number of nickels (n) that come in.

\[ A(n) = 0.05n \]

We say: "A of n is equal to 0.05n"

#### Ex1)

Write the equation \( y = 2x - 5 \) in function notation.

**Label:** independent/dependent, domain/range, input/output, the variables

\[ \text{Out} \rightarrow \text{Dep} \]

\[ \text{Range} \rightarrow \text{Domain} \in \text{lin} \]

\( y \) depends on \( x \), so \( y \) is a function of \( x \) and we write

\[ f(x) = 2x - 5 \]

\( f(x) \) is like \( y \)

Note: we can use letters other than \( f \) such as \( g, h, k \)

\[ g(x) = 2x - 5, \text{ etc.} \]

Note: we can work in the opposite direction by changing function notation back into the more familiar equations in 2 variables, e.g.

\[ g(x) = 3x + 4 \]

\( y = 3x + 4 \)
Ex2) The equation \( C = 23n + 550 \) represents the cost (\( C \)) of a banquet where \( n \) people attend.

a) Describe the function. The cost (\( C \)) depends on the number of people who come (\( n \)). \( C \) is a function of \( n \).

b) Write the function in function notation.
\[
C(n) = 23n + 550
\]

c) Find \( C(100) = \) ___ and explain what this represents.
\[
\begin{align*}
C(n) &= 23n + 550 \\
C(100) &= 23(100) + 550 \\
C(100) &= 2300 + 550 \\
C(100) &= 2850
\end{align*}
\]

Sub in \( n = 100 \).

When \( n = 100 \) (100 people) \( C = 2850 \) (cost is \( \$2850 \)).

\( C(n) = 23n + 550 \)

4000 = 23n + 550

Solve for \( n \).

\[
\begin{align*}
\frac{4000 - 550}{23} &= \frac{3450}{23} \\
150 &= \frac{23n}{23}
\end{align*}
\]

\( n = 150 \)

d) Find \( n \) for \( C(n) = 4000 \) and explain what this represents.

Replace \( C(n) \) with 4000.

When the cost is 4000 (\( \$4000 \)), there were 150 people (\( n = 150 \)).

Ex3) For the function \( f(x) = 3x - 4 \)

a) Write as a 2-variable equation.
\[
y = 3x - 4
\]

b) Determine the values of \( f(6), f(4), f(-2) \) (Sub in \( x = \) ___)
\[
\begin{align*}
f(6) &= 3(6) - 4 \\
f(6) &= 18 - 4 \\
f(6) &= 14
\end{align*}
\]

\[
\begin{align*}
f(4) &= 3(4) - 4 \\
f(4) &= 12 - 4 \\
f(4) &= 8
\end{align*}
\]

\[
\begin{align*}
f(-2) &= 3(-2) - 4 \\
f(-2) &= -6 - 4 \\
f(-2) &= -10
\end{align*}
\]
\( x = 6, \ y = 14 \)
\( x = 4, \ y = 8 \)
\( x = -2, \ y = -10 \)

c) Determine the value of \( x \) for \( f(x) = 2 \) and for \( f(x) = -1 \) (Sub in \( y = \) ___)
\[
\begin{align*}
f(x) &= 3x - 4 \\
f(x) &= 3x - 4 \\
f(x) &= 3x - 4
\end{align*}
\]

\[
\begin{align*}
2 &= 3x - 4 \\
2 &= 3x - 4 \\
2 &= 3x - 4
\end{align*}
\]

\[
\begin{align*}
+4 &= +4 \\
+4 &= +4 \\
+4 &= +4
\end{align*}
\]

\[
\begin{align*}
\frac{6}{3} &= \frac{3x}{3} \\
2 &= x
\end{align*}
\]

\( x = 2, \ y = 2 \)
\( x = 1, \ y = -1 \)

Reflection: For example 2 about the banquet, what values of \( n \) do not make sense as possible domain values? (Look back: what does \( n \) represent?)

You can't have negative \( n \) (negative # of people) or decimals/fractions of people. \( n \) must be a whole number (0, 1, 2, ...).
5.3/5.4 – Interpreting and Drawing Graphs

Goal: to practice interpreting graphs and to practice drawing graphs (working back and forth between situations and their matching graphs)

Toolkit:
- anything you remember about graphs
- indep/dep. variables
- interpreting word problems
- horizontal vs vertical

Main Ideas:

"Try This" p. 277

<table>
<thead>
<tr>
<th>Work with a partner on the “Try This” on page 277</th>
</tr>
</thead>
<tbody>
<tr>
<td>A OA fill tub</td>
</tr>
<tr>
<td>B AB tub full for 50 min</td>
</tr>
<tr>
<td>C BC person gets in</td>
</tr>
<tr>
<td>D CD person stays in about 10 min</td>
</tr>
<tr>
<td>E DE person gets out</td>
</tr>
<tr>
<td>F EE tub has water/no person</td>
</tr>
<tr>
<td>G FG tub drains</td>
</tr>
</tbody>
</table>

Ex1) Label key information on the following graph. When/how is it increasing? Decreasing?

Ex1) Label key information on the following graph. When/how is it increasing? Decreasing?

Dependent Variable

Independent Variable

more steep = faster  
more shallow = slower
Ex2) Interpret Graph

Ex2) Using the graph, EXPLAIN the answer to each question:

a) Who is the oldest? How old is s/he?
   - F is oldest at 17 yrs  
   (farthest to right - age)

b) Who is the youngest? How old is s/he?
   - A is youngest at 0 yrs  
   (farthest left - age)

c) Who has the same height?
   - What is that height?
   - E & F have same height (160 cm)

d) Who has the same age? What is that age?
   - C & D are same age (8 years)

e) Which person is taller for his/her age: person E or F?
   - E is taller for his/her age than F  
   (same height, but E is younger)

f) What are the coordinates (ordered pairs) for persons C and D?
   - (8, 120) and (8, 140)

9) Is this a function?  No - because domain values are repeated (8, 120) (8, 140)  
   when first value is 8, 2nd value could be  
   2 different things.  
   X not a function.
Ex3) Use the graph to answer the following questions and to describe the journey for each segment of the graph.

a) How far is it from Victoria to Nanaimo?
   112 km

b) Where do you start the day trip? End it?
   Start: Victoria
   End: Victoria

c) Which is the independent variable? The Dependent?
   Indep: Time
   Dep: distance

d) Fill in the following chart:

<table>
<thead>
<tr>
<th>Segment</th>
<th>Graph</th>
<th>Journey</th>
</tr>
</thead>
<tbody>
<tr>
<td>OA</td>
<td>The graph goes up to the right, so as time increases, the distance from Victoria increases.</td>
<td>For the first 30 min, we travel about 80 km toward Nanaimo.</td>
</tr>
<tr>
<td>AB</td>
<td>The graph is horizontal, so as time increases, the distance from Victoria stays the same.</td>
<td>For about 15 min, we do not move farther from or closer to Victoria. Must be stopped! (Pics? Snack?)</td>
</tr>
<tr>
<td>BC</td>
<td>Graph goes up to right, so time increases, distance from Victoria increases.</td>
<td>The car travels approximately 80 km toward Nanaimo and arrives in Nanaimo. (takes about 1 h 15 min)</td>
</tr>
<tr>
<td>CD</td>
<td>Graph is horizontal (distance stays same)</td>
<td>We spend 3 hours in Nanaimo.</td>
</tr>
<tr>
<td>DE</td>
<td>The graph goes down to the right, so as time increases, the distance decreases</td>
<td>The car takes 2 h to return to Victoria.</td>
</tr>
</tbody>
</table>
Ex4) Situation → graph

Ex4) SKETCH a graph to represent the following situation:

Our classroom has different classes in it throughout the day. Different classes prefer the room to be warmer or colder. During the first block of the day, the classroom is too cold at 16°C. We turn up the heat at 8:30 to 20°C. By lunch, the heat outside and the bodies in the room have made the classroom too hot, at 22°C. We turn the heat down to 18°C. At 4:00, the school shuts off the heat and the room returns to the same temperature it was first thing this morning.

Start your graph at 8:00am and end it at 5:00pm.

You need to include:
- Title
- Axis labels (with units)
- Smart scales (how far apart you space the #s)
- Label the segments (OA, AB, etc.)

8:00 - 8:30 16°C
8:30 + warms to 20°C
11:30, warms to 22°C
11:30 + cool down to 18°C
4:00 cool to 16°C
5:00 should be about 16°C

Note: need time to warm and cool

Note: "Sketch" means it's ok if it's a rough idea or includes estimates.

Reflection: Why are the points joined on the graphs in Examples 1, 3, 4 but NOT joined in the graph in Example 2?

* Discuss today or next class *

Ex2 - the graphs show different people, not one person changing overtime.
Ex1,3,4 we connect the points because it makes sense to have decimal values between the points on the graph.
Goal: to examine the properties of graphs of relations and graphs of functions

Toolkit:
- Discrete vs Continuous
- graphing \((x,y)\) on the coordinate plane
- functions, domain, range

Main Ideas:

Definitions
Function – a function has ordered pairs with different first coordinates (see VLT below)

Domain – the domain is the set of values of the independent variable \((x-\text{axis})\) [first element]

Range – the range is the set of values of the dependent variable \((y-\text{axis})\) [second element]

Discrete – (dots) The spaces between points on the graph have no literal meaning (e.g. you can’t have 1.4 people)

Continuous – (connect the dots) The spaces between points have meaning (e.g. 1.4 seconds occurs between 1 second and 2 seconds, and something is happening then)

Warm-up: consider the relation that associates every natural number with its double

As a table of values:

<table>
<thead>
<tr>
<th>Natural number ((x))</th>
<th>Double the number ((y))</th>
<th>((x,y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>(1, 2)</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>(2, 4)</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>(3, 6)</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>(4, 8)</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>(5, 10)</td>
</tr>
<tr>
<td>etc.</td>
<td>etc.</td>
<td></td>
</tr>
</tbody>
</table>

(1, 2) Start at \((0,0)\)
over 1 right, up 2

domain value is 1, range value is 2

As a formula:

\[ y = 2x \]

As a graph:

What is the domain value if the range value is 8?
\[ x = 4 \] \[ y = 8 \]
Check: \[ 4 \times 2 = 8 \]
Functions

Is the relation in the warm-up a FUNCTION? How can we tell?

Yes- no repeated x-values (domains)

Vertical Line Test - (VLT) - A graph represents a function when no two points on the graph lie on the same vertical line. (Run a ruler vertically along graph; should only ever touch once.)

Non-functions

What if it is not a function? We can still call it a relation

Graph the table of values

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>x₁</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

Ex 1)

Ex 1) State whether each relation is a FUNCTION (yes or no) and whether it is discrete or continuous.

a) Function? Yes No

Discrete / Continuous / Connected

b) Function? Yes No

Discrete / Continuous / Connected

c) Function? Yes No

Discrete / Continuous / Dots

Ex 2)

Ex 2) EXPLAIN whether the graph for each situation should be discrete or continuous.

a) The amount of money charged to your online music account is a function of the number of songs you download.
   Cannot buy 2.7 songs, or 1.45 songs.
   Discrete!

b) The amount of water in a bathtub is a function of time passing as it is filled, emptied, etc.
   Time continues (0.2 seconds, 0.3 seconds, 0.4, . . )
   Continuous!

Reflection: Return to your Frayer model from 5.2 and add anything you wish to. What are ALL the ways we have so far of recognizing a function?

- no repeated x's (domain values / first element)
- only one arrow leaving each element in first bubble of arrow diagram
- passes VLT (graph touches any vertical line only once)
5.5B – Domain and Range

Goal: to determine (and express mathematically) the domain and range of graphs and other relations

Toolkit: Inequality Signs

> is greater than
< is less than
≥ is greater than or equal to
≤ is less than or equal to
< is Like an L for Left/Less than/Lower than
> is the other one

Main Ideas:

Review

Write an inequality that is represented by each graph.

○ = point not included (> or <)    ● = point included (≥ or ≤)

\[ x > -1 \]

\[ x \leq 3 \]

\[-3 \leq x < 0.5\]

"\( x \) is between -3 and 0.5.
not included \( \leq \)
smaller # \( \leq x \leq \) larger #

Domain and Range

The domain is the set of all \( x \) values (so we’ll use the \( x \)-axis to help us)
The range is the set of all \( y \) values (so we’ll use the \( y \)-axis to help us)

Ex1) State the domain and range for this relation.

\[ \{ (-3,0), (-2,0), (0,1), (1,3), (3,3), (4,5) \} \]

Domain: \[ \{-3, -2, 0, 1, 3, 4\} \]
Range: \[ \{0, 1, 3, 5\} \]

Hint: For discrete graphs with only a few points, list their coordinates (ordered pairs), then list all the first coordinates (x) for the domain, and second (y) for range.
Ex2) For a **continuous** relation, we cannot describe every single x-value or y-value (there are infinitely many!).

Since we can't list ALL the domain values or ALL the range values, it helps to think about "minimum" and "maximum" values:

**Domain:**
- How far **left** does the graph go? (min) \(-2\)
- How far **right**? (max) \(2\)

**Range:**
- How far **down** does the graph go? (min) \(0\)
- How far **up**? (max) \(2\)

There are 4 different ways to state domain and range:

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>In words:</strong> All real numbers between -2 and 2, including -2 but not 2.</td>
<td><strong>In words:</strong> All real numbers between 0 and 2 inclusive.</td>
</tr>
<tr>
<td><strong>Number Line:</strong></td>
<td><strong>Number Line:</strong></td>
</tr>
<tr>
<td><img src="image1.png" alt="Number Line" /></td>
<td><img src="image2.png" alt="Number Line" /></td>
</tr>
<tr>
<td><strong>Interval Notation:</strong> ([-2, 2))</td>
<td><strong>Interval Notation:</strong> ([0, 2])</td>
</tr>
<tr>
<td><strong>Set Notation:</strong> ({x</td>
<td>-2 \leq x &lt; 2, x \in \mathbb{R}})</td>
</tr>
</tbody>
</table>

Remember: \(\mathbb{R}\) is "all real numbers"
Ex3) State the domain and range for each relation

a) 

\[ \text{Domain:} \quad x \geq 2 \]
\[ \{ x | x \geq 2, x \in \mathbb{R} \} \]
\[ [2, \infty) \]

\[ \text{Range:} \]
\[ y \geq 1 \]
\[ \{ y | y \geq 1, y \in \mathbb{R} \} \]
\[ [1, \infty) \]

b) 

\[ \text{Domain:} \]
\[ x \text{ is all real #s} \]
\[ x \in \mathbb{R} \]
\[ \{ x | x \in \mathbb{R} \} \]
\[ (-\infty, \infty) \]

\[ \text{Range:} \]
\[ y \text{ is all real numbers} \]
\[ y \text{ is greater than 1 or equal to 1} \]
\[ \{ y | y \geq 1, y \in \mathbb{R} \} \]
\[ [1, \infty) \]

Reflection: Describe in words how you would find the domain and range of a function using its graph. You may wish to use a specific graph to help explain.
- Consider min/max of \( x \) (left/right) and \( y \) (up/down)
- Choose a notation that makes most sense to me!
5.6 – Properties of Linear Relations

Goal: to identify and represent linear relations in different ways

Toolkit:
- Independent Variable changes ($x$)
- Dependent Variable relies on indep. var. ($y$)
- Constant = an unchanging number (not a variable)
- Reducing fractions
- Anything you remember about linear relations!

Main Ideas:

Warm-up: During a certain stretch of a road trip, you set your cruise control and start a timer (at zero) and reset your trip-meter to zero. Your friend watches to see how many kilometers you’ve gone after 30 minutes, one hour, an hour and a half, etc. and she keeps track of it in a table.

a) Identify the independent $\rightarrow$ time and dependent $\rightarrow$ distance variables

b) Graph the data from the table

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Distance (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>45</td>
</tr>
<tr>
<td>1</td>
<td>90</td>
</tr>
<tr>
<td>1.5</td>
<td>135</td>
</tr>
<tr>
<td>2</td>
<td>180</td>
</tr>
<tr>
<td>2.5</td>
<td>225</td>
</tr>
<tr>
<td>3</td>
<td>270</td>
</tr>
</tbody>
</table>

$+\,5$ $+\,5$ $+\,5$

$\begin{array}{c}
0 \\
0.5 \\
1 \\
1.5 \\
2 \\
2.5 \\
3
\end{array}$

$\begin{array}{c}
0 \\
45 \\
90 \\
135 \\
180 \\
225 \\
270
\end{array}$

$\Rightarrow 90 \text{ km/h} \Rightarrow \text{speed!}$

For a linear relation, a constant change in the independent variable results in a constant change in the dependent variable.

(Hint: make sure to list independent variable ($x$) values in numerical order!)
Ex 1) Recognizing a linear relation in table form

Ex 1) Which tables of values represent linear relations? Identify the independent and dependent variables for each relation and if linear, find the rate of change.

rate of change = \( \frac{\text{change in DEP.}}{\text{change in INDEP.}} \)

a) Temperatures in Celsius (C) and Fahrenheit (F)

<table>
<thead>
<tr>
<th>C</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>32</td>
</tr>
<tr>
<td>5</td>
<td>41</td>
</tr>
<tr>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>15</td>
<td>59</td>
</tr>
</tbody>
</table>

constant change \( +5 \)

rate of change = \( \frac{9}{5} \) \( \checkmark \) linear

b) Number of bacteria \( (n) \) growing on an old sandwich after \( t \) minutes

<table>
<thead>
<tr>
<th>t</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>10</td>
<td>24</td>
</tr>
<tr>
<td>15</td>
<td>48</td>
</tr>
<tr>
<td>20</td>
<td>96</td>
</tr>
</tbody>
</table>

constant change \( +5 \)

\( +6 \) \( +12 \) \( +24 \) \( +48 \) \( \checkmark \) nonlinear

c) The amount of HST (T for tax) charged on different purchases of Amount (A)

<table>
<thead>
<tr>
<th>A</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>1.80</td>
</tr>
<tr>
<td>30</td>
<td>3.60</td>
</tr>
<tr>
<td>45</td>
<td>5.40</td>
</tr>
<tr>
<td>60</td>
<td>7.20</td>
</tr>
<tr>
<td>75</td>
<td>9.00</td>
</tr>
</tbody>
</table>

constant change \( +15 \)

rate of change = \( \frac{1.80}{15} \) \( \checkmark \) linear

d) How else could we determine whether these tables of values represent linear relations?

Graphing! Plot the values: do they form a line?

e) Below are the equations for each table of values. What do you notice about the equations of the linear relations? Relate the equation to what you know from the table of values.

a) \( F = \frac{9}{5} C + 32 \)

rate of change first dep. var. in table

b) \( n = 2^t \)

not linear exponents

c) \( T = 0.12A + 0 \)

rate of change

\[ \text{letter} = \# \times \text{letter} + \# \]

\[ \text{dep. var} = \text{rate of change (indep.) + constant} \]

(from table - dep. paired with 0)
Ex2) Recognizing a linear relation in equation form

Create a table of values for each equation, then graph it and decide whether it is a linear relation.

2 points enough (always makes a line)

How many points do you **NEED** to tell whether a relation is linear? **3** (minimum)

a) \( y = -2x + 8 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>12</td>
</tr>
<tr>
<td>-1</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

\( f(x) = -2x + 8 \)

\( f(-2) = -2(-2) + 8 = 4 + 8 = 12 \)

\( f(-1) = -2(-1) + 8 = 2 + 8 = 10 \)

\( f(0) = -2(0) + 8 = 0 + 8 = 8 \)

\( f(1) = -2(1) + 8 = -2 + 8 = 6 \)

\( f(2) = -2(2) + 8 = -4 + 8 = 4 \)

\( f(3) = -2(3) + 8 = -6 + 8 = 2 \)

Connect dots (why?)

You choose (pick some) \( \sqrt{\text{Linear}} \)

b) \( y = 3x^2 - 3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>9</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
</tbody>
</table>

\( f(x) = 3(x)^2 - 3 \)

\( f(-2) = 3(-2)^2 - 3 = 12 - 3 = 9 \)

\( f(-1) = 3(-1)^2 - 3 = 3 - 3 = 0 \)

\( f(0) = 3(0)^2 - 3 = 0 - 3 = -3 \)

\( f(1) = 3(1)^2 - 3 = 3 - 3 = 0 \)

\( f(2) = 3(2)^2 - 3 = 12 - 3 = 9 \)

\( \sqrt{\text{Non-Linear}} \)

C) \( y = 6 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

Horizontal line at a height of 6 \( y = 6 \)

\( \sqrt{\text{Linear}} \)

d) \( x = 3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Vertical line along \( x \)-axis at 3 \( x = 3 \)

\( \sqrt{\text{Linear}} \)
Ex3) Sort the equations we have seen so far by crossing out all NON-linear relations. How can we recognize linear relations without graphing?

\[ F = \frac{9}{5}C + 32 \quad n = 6(2)^x \quad T = 0.12A + 0 \]

\[ y = -2x + 8 \quad y = 3x^2 - 3 \quad y = 6 \quad x = 3 \]

<table>
<thead>
<tr>
<th>Dep. Var.</th>
<th>Indep. Var.</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial value</td>
<td>rate</td>
<td>(no exponents, weird operations!)</td>
</tr>
</tbody>
</table>

Ex4) A banquet hall costs $80 to rent, and it costs $30 per person for catering. Write an equation to represent the total cost of the banquet (C) in relation to the number of people who attend (n).

\[ C = 30n + 80 \]

As \( n \) (#people) goes up by 1, cost goes up by 30.
80 is initial cost.

Ex5) Determine and explain the rate of change using the graph of the linear relation.

Step 1) Find the dependent (height) and independent variables (time).

Step 2) Find two EASY TO READ points.

Step 3) Find the change in height (y, dep. var.) and the change in left/right (x, indep. var.)

\[ \frac{\Delta y_{\text{dep var}}}{\Delta x_{\text{indep var}}} = \frac{\text{down 2}}{\text{right 4}} = \frac{-2}{4} = -\frac{1}{2} \text{ m/s} \]

Step 4) Reduce the fraction and pay attention to units to help see what the rate represents.

\[-\frac{1}{2} = -0.5 \text{ m/s} \]

The flag drops at 0.5 metres per second.

Reflection: Compare (similarities and differences) how you find the rate of change for a table of values versus a graph.

Rate is still \( \frac{\text{change in dep}}{\text{change in indep}} \), but in table you might do \( \frac{\text{right column change}}{\text{left column}} \) (count up or down).

On graph: \( \frac{\Delta y_{\text{axis}}}{\Delta x_{\text{axis}}} \) (count up/down and over).
5.7 – Interpreting Graphs of Linear Functions

Goal: to use intercepts, rate of change, domain, and range to describe the graph of a linear function.

Toolkit:
- linear relations so far
- finding rates of change
- substitution & solving

Main Ideas:

Warm-up: True or False? A linear relation is always a linear function. 

False: vertical lines (x = constant) are linear relations but fail the vertical line test (not functions)

What is a horizontal intercept?
Where graph crosses horizontal axis (often x-axis)
Vert. int. at 1 (0, 1)
Vertical axis

Occurs when x = 0

or other indep. var.

A vertical intercept? *also "initial value"!

Where graph crosses vertical axis (often y-axis)
Hor. int. at 2 (2, 0)

Occurs when y = 0

or other dep. var.

Ex1) Determining features of a linear function’s graph

Ex1) What are some of the key features of this graph?

a) Write the coordinates of the points where the graph intersects the axes.

(0, 1600) (40, 0)

b) Determine the vertical and horizontal intercepts.

Vert. int. at 1600
Hor. int. at 40

c) Describe what the points of intersection represent. Here, ...

Vert. int. means the Volume of the tub when t = 0 (at start)
Hor. int. is time when volume is 200 (tub empty).

d) What are the domain and range of this function?

D: 0 ≤ t ≤ 40
R: 0 ≤ V ≤ 1600

e) What is the rate of change for this function?

Vol/ops 800 L
in 40 min time

\[ \frac{-800 \text{ L}}{40 \text{ min}} = -20 \text{ L/min} \]
Ex2) Sketching a graph using function notation and intercepts

Step 1: Determine the y-intercept

\[ f(x) = 2x - 4 \]

\[ f(0) = 2(0) - 4 \]

\[ f(0) = -4 \]

or \( y = -4 \) \( (0, -4) \)

when \( x = 0 \) \( y = -4 \)

Step 2: Determine the x-intercept

\[ f(x) = 2x - 4 \]

\[ 0 = 2x - 4 \]

\[ x = 2 \]

\[ y = 2 \]

when \( y = 0 \) \( x = 2 \) \( (2, 0) \)

Ex3) Which graph has a rate of change of \( \frac{-3}{2} \) and a vertical intercept of 6?

- **A**

Rate: down 2

right 3

\[ \frac{\text{down} 2}{\text{right} 3} = \frac{-2}{3} \]

a) Using the correct graph, what is the distance when time is 2?

When \( t = 2 \), follow dotted line: \( d = 3 \)

b) Using the correct graph, what is the time is it when the distance is 1?

When \( d = 1 \), follow 

\[ d = \frac{-3}{2} t + 6 \]

\[ 1 = \frac{-3}{2} t + 6 \]

\[ \frac{-11}{2} = \frac{-3}{2} t \]

\[ t = \frac{-11}{3} \]

\[ t = 3.6 \]

Reflection: Describe how you can tell from a graph whether a linear function has a positive or negative rate of change.

- **Up** from left to right: \( \Theta \) (we read left to right)
- **Down** from left to right: \( \Theta \)